

# Computational Insights and the Theory of Evolution

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# Evolution Before Darwin

- Erasmus Darwin



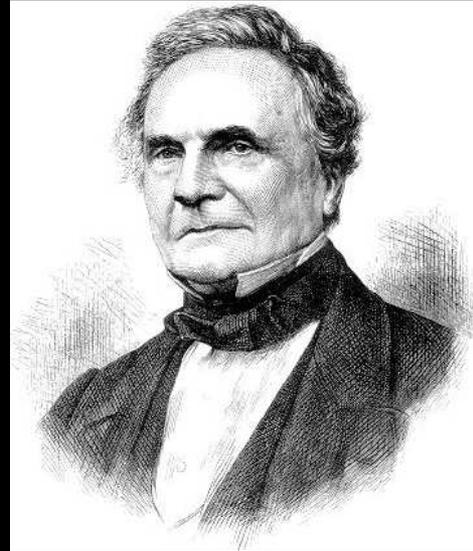
# Before Darwin

- J.-B. Lamarck



# Before Darwin

- Charles Babbage



[Paraphrased]

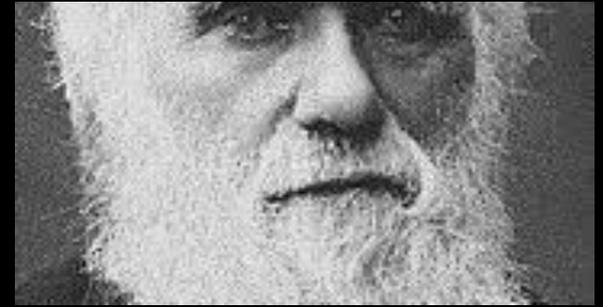
*“God created not species, but the Algorithm for creating species”*

# Darwin, 1858



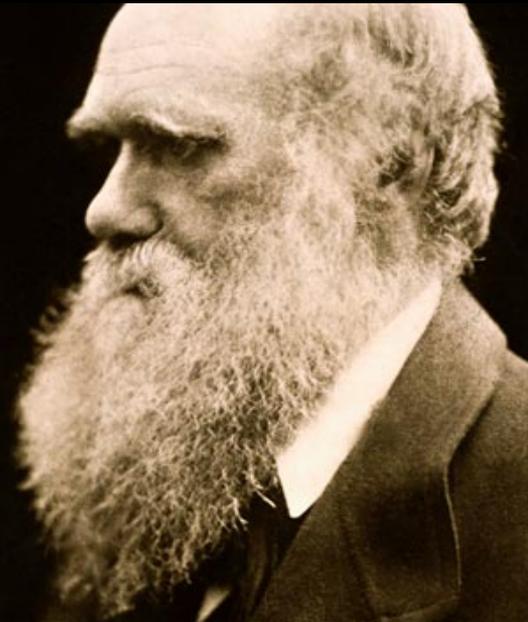
- Common Ancestry
- Natural Selection

# *The Origin of Species*



- Possibly the world's most masterfully compelling scientific argument
- The six editions 1859, 1860, 1861, 1866, 1869, 1872

# The Wallace-Darwin papers



Brilliant argument, and yet many questions left unasked, e.g.:

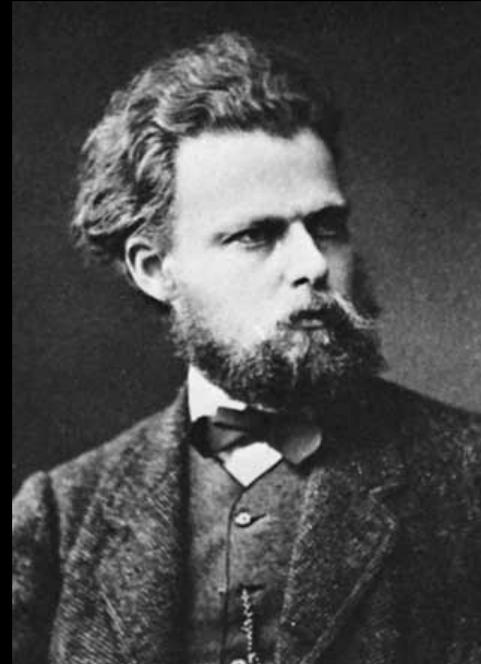
- How does novelty arise?
- What is the role of sex?

# After Darwin

- A. Weismann

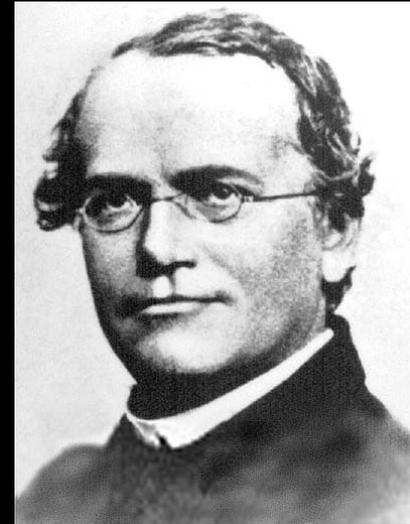
[Paraphrased]

*“The mapping from genotype to phenotype is one-way”*



# Genetics

- Gregor Mendel [1866]
- Number of citations  
between 1866 and 1901:



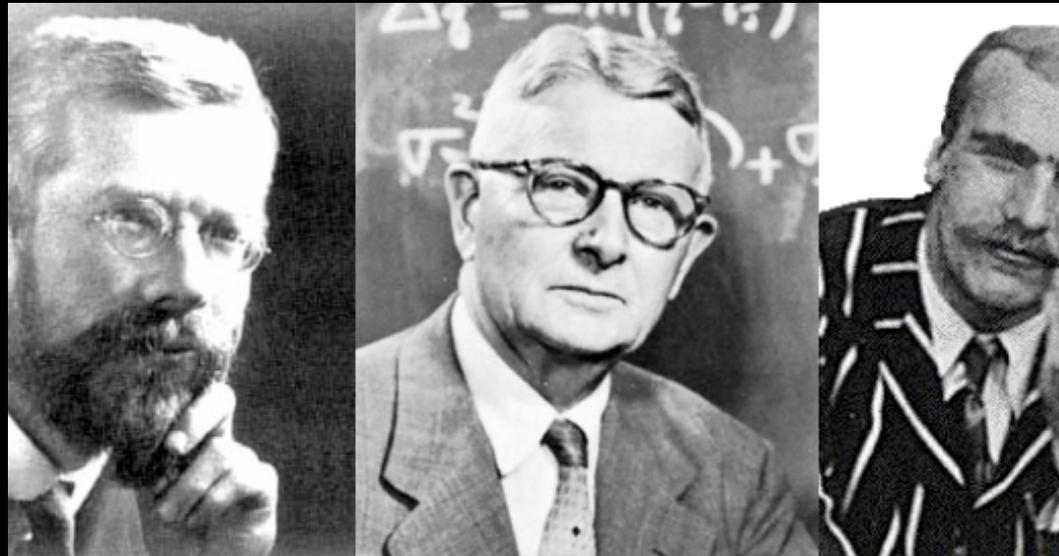
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# The crisis in Evolution

## 1900 - 1920

- Mendelians vs. Darwinians
- Geneticists vs. Biometricists/  
Gradualists
- Population genetics

# The “Modern Synthesis” 1920 - 1950



Fisher – Wright - Haldane

# Big questions remain

e.g.:

- How does novelty arise?
- What is the role of sex?

# Evolution and Computer Science



*How do you find a 3-billion long string in 3 billion years?"*

**L. G. Valiant**

At the Wistar conference (1967), Schutzenberger asked virtually the same question

# Valiant's Theory of the Evolvable

- Which functions (traits of an organism) can evolve by natural selection?
- Properly formalized, this question leads to identifying obstacles to evolution
- For example, the function has to be learnable (actually, statistically so)
- Evolvability is a (quite) restricted form of learnability

# Evolution and CS Practice: Genetic Algorithms [ca. 1980s]

- To solve an optimization problem...
- ...create a population of solutions/genotypes
- ...who procreate through sex/genotype recombination...
- ...with success proportional to their objective function value
- Eventually, some very good solutions are bound to arise in the soup...

# And in this Corner...

## Simulated Annealing

- Inspired by *asexual* reproduction
- Mutations are adopted with probability increasing with fitness/objective differential
- ... (and decreasing with time)

# The Mystery of Sex Deepens

- Simulated annealing (asexual reproduction) works fine
- Genetic algorithms (sexual reproduction) don't work
- In Nature, the opposite happens: Sex is successful and ubiquitous



# A Radical Thought

- What if sex is a mediocre optimizer of fitness (= expectation of offspring)?
- What if sex optimizes something else?
- And what if this something else is its *raison d'être*?

# Mixability!

- In a recent paper [LPDF, PNAS 2008] we establish through simulations that:
- Natural selection under asex optimizes fitness
- But under sex it optimizes *mixability*:
- The ability of alleles (gene variants) to perform well with a broad spectrum of other alleles

# Explaining Mixability

- Fitness landscape of a 2-gene organism

Rows: alleles of gene A

3	2	4	5	4
1	0	0	7	2
2	1	0	4	3
1	8	1	3	2

Columns: alleles of gene B

Entries: fitness of the combination

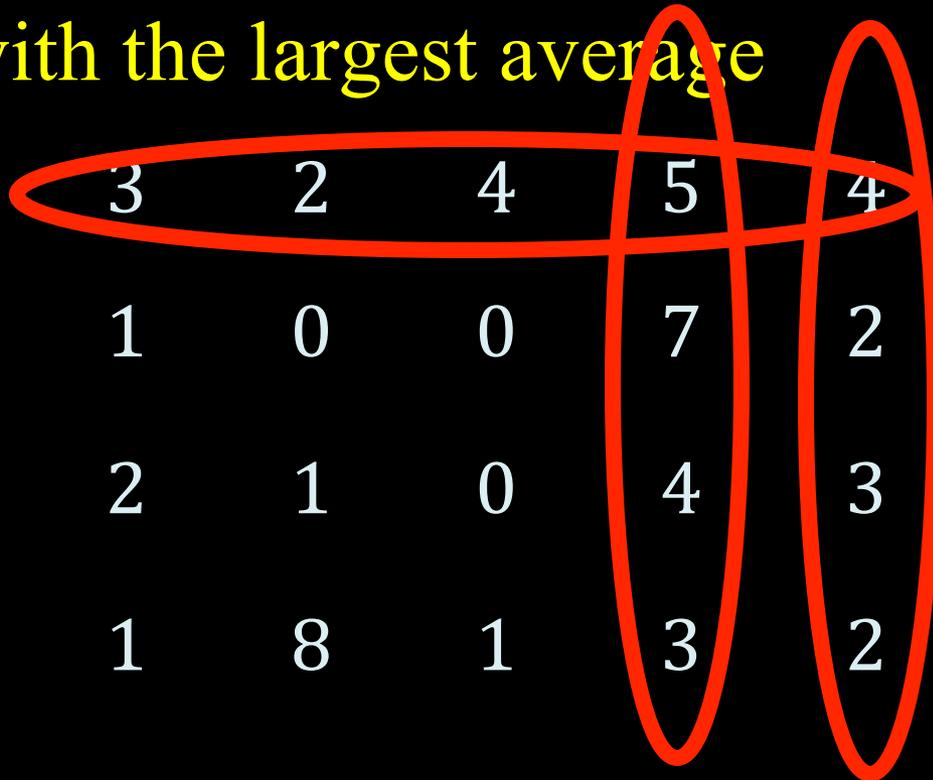
# Explaining Mixability (cont)

- Asex will select the largest numbers

3	2	4	5	4
1	0	0	7	2
2	1	0	4	3
1	8	1	3	2

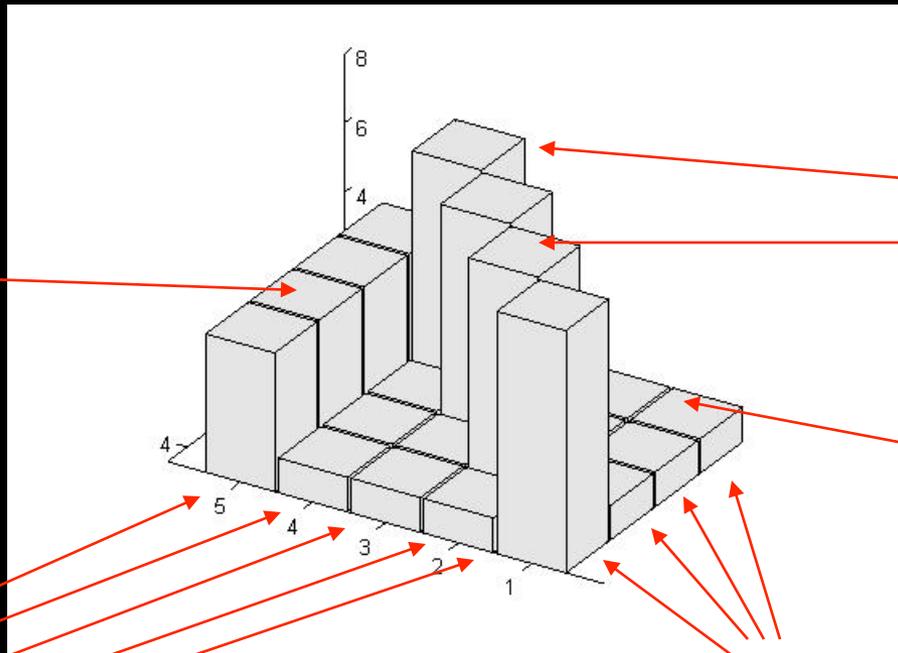
# Explaining Mixability (cont)

- But sex will select the rows and columns with the largest average



3	2	4	5	4
1	0	0	7	2
2	1	0	4	3
1	8	1	3	2

# In Pictures



“plateau”

peaks

troughs

alleles  
(variants)  
of gene A

alleles  
of gene B

# Sex favors plateaus over peaks

**Theorem** [Livnat, P., Feldman 11] In landscapes of this form

- Unless  $\text{peak} > 2 \times \text{plateau}$ , in sexual reproduction the plateau will dominate and the peaks will become extinct
- In asexual reproduction, the peaks will always dominate and the plateau will become extinct

# And plateaus accelerate evolution

- They act as springboards allowing alternatives to be explored *in parallel*...
- ...and this acceleration promotes *speciation* (the creation of new species)...
- ...which results in an altered landscape...
- ...in which sex selects more plateaus...
- ...and life goes on...

# Pointer Dogs



# Pointer Dogs

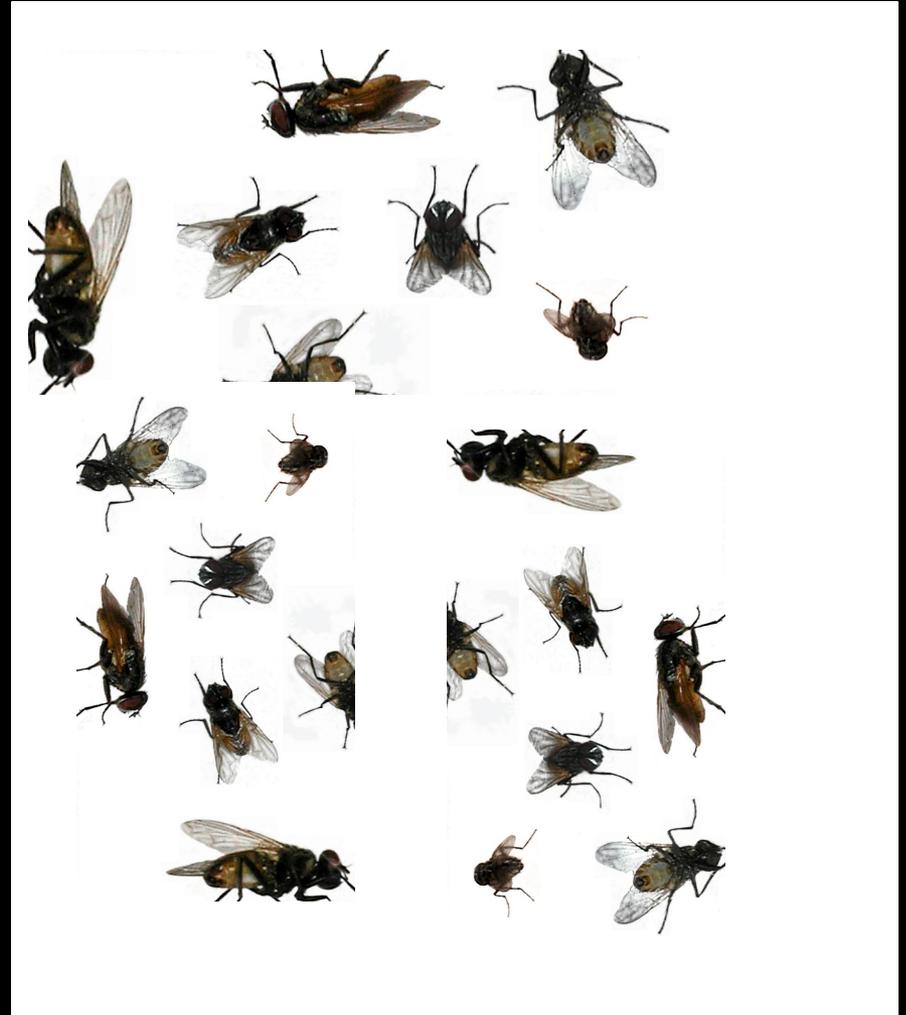


C. H. Waddington

# Waddington's Experiment (1952)

Generation 1

Temp: 20° C



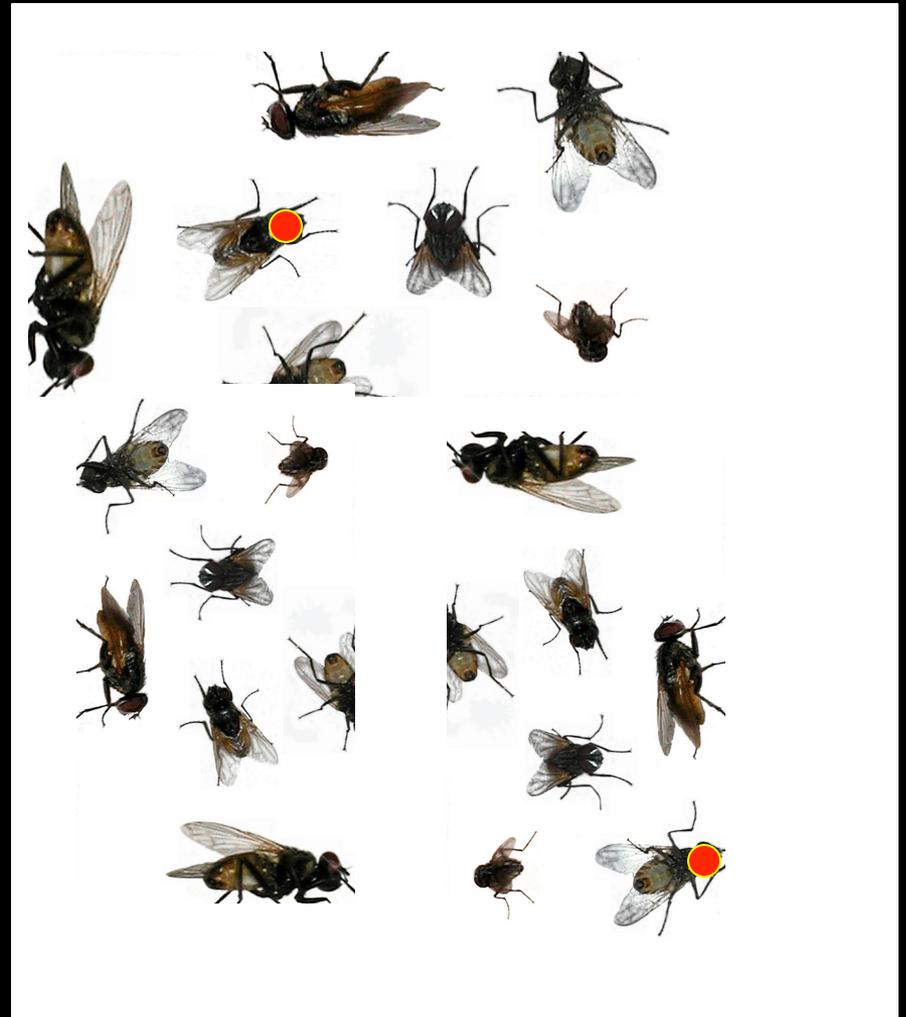
# Waddington's Experiment (1952)

Generation 2-4

Temp: 40° C

~15% changed

Select and breed those



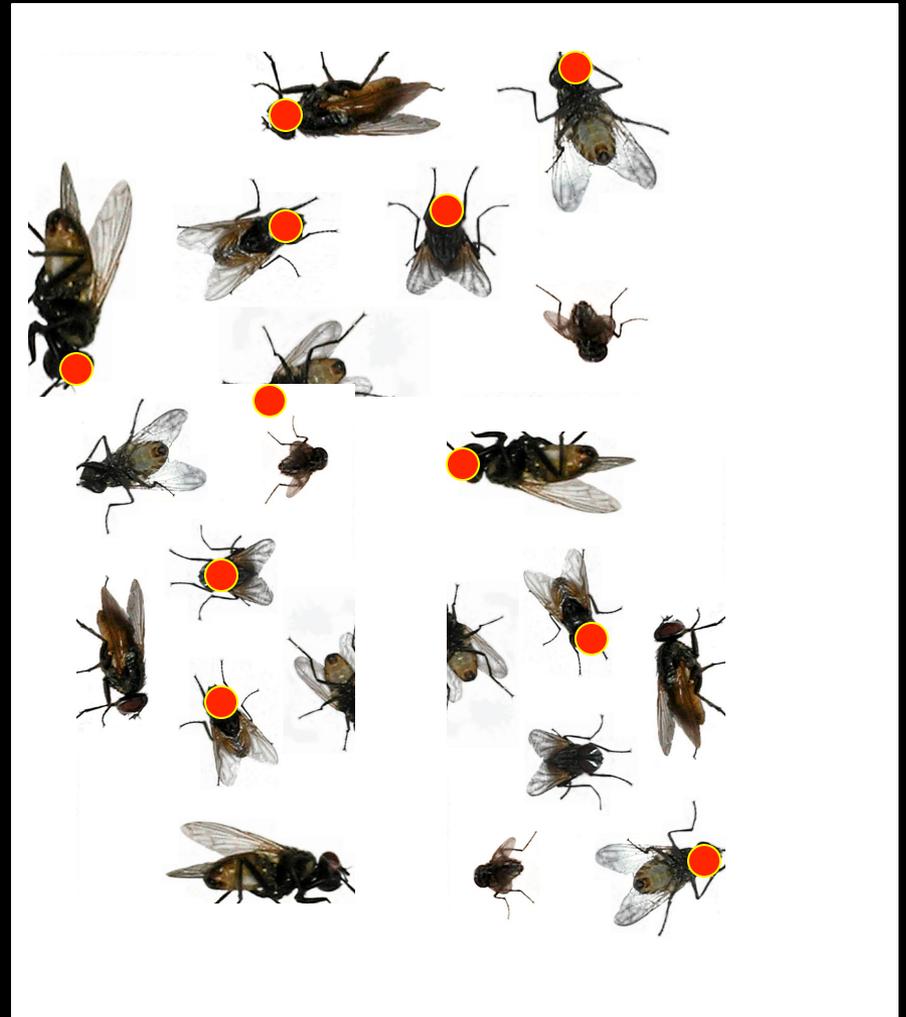
# Waddington's Experiment (1952)

Generation 5

Temp: 40° C

~60% changed

Select and breed those



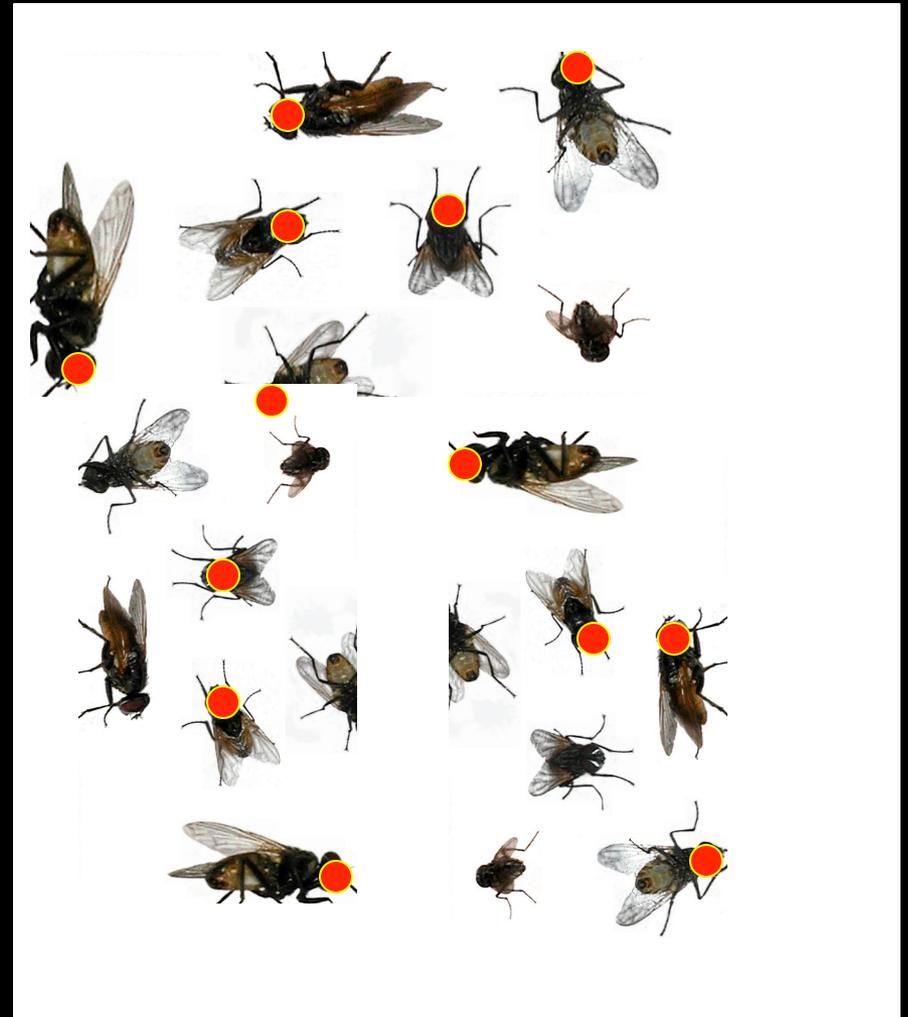
# Waddington's Experiment (1952)

Generation 6

Temp: 40° C

~63% changed

Select and breed those



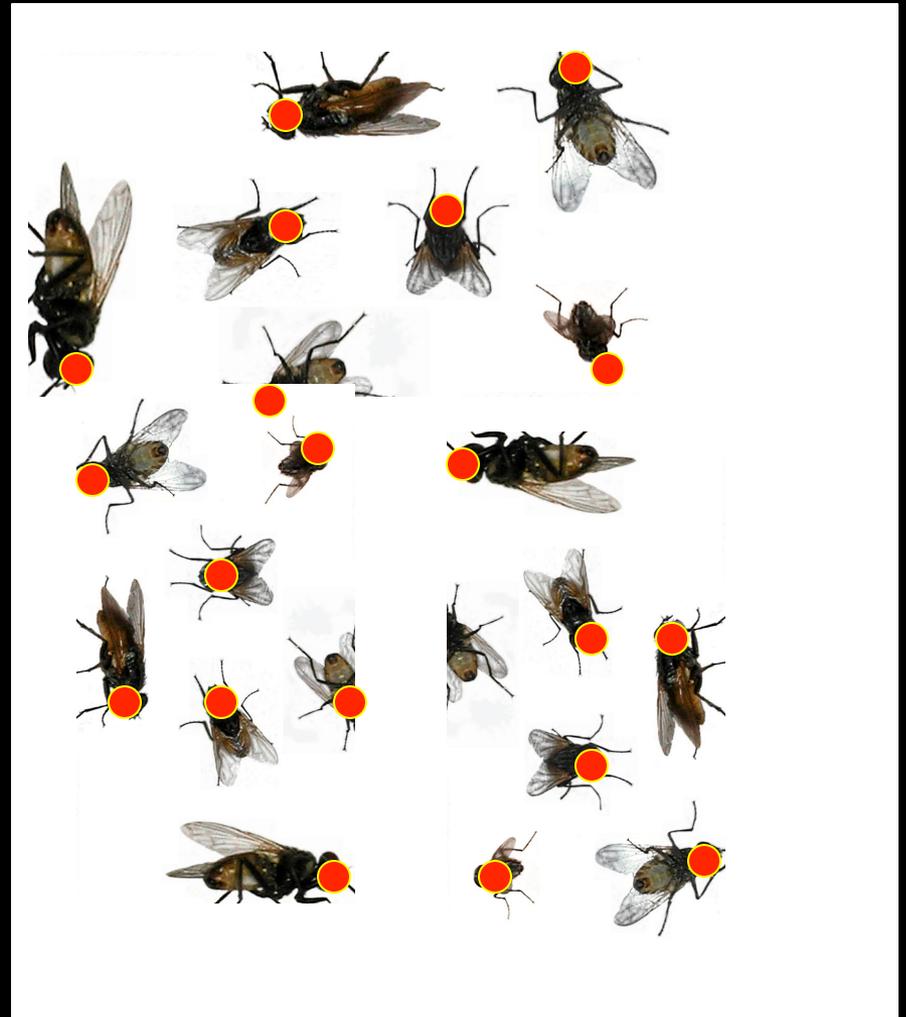
# Waddington's Experiment (1952)

(...)

Generation 20

Temp: 40° C

~99% changed

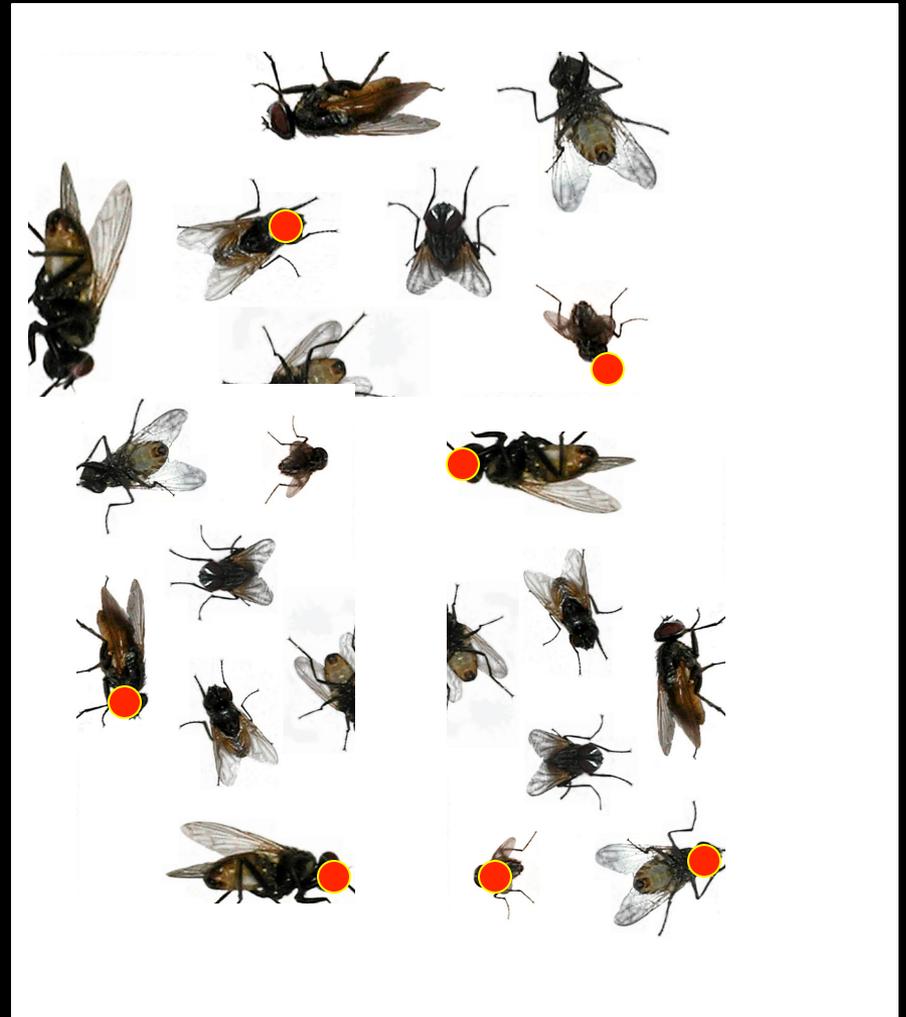


# Surprise!

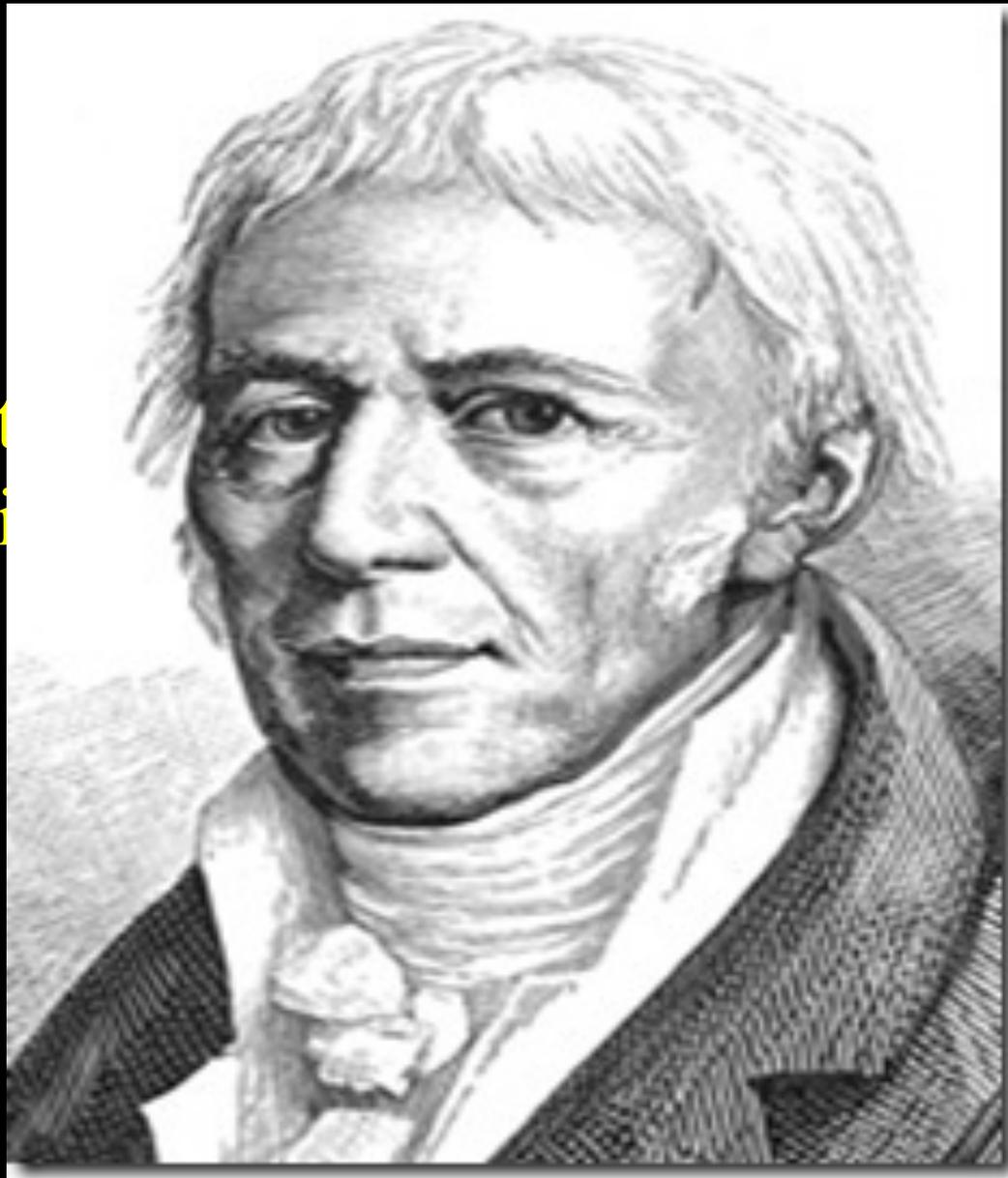
Generation 20

Temp: 20° C

~25% stay changed!!



- Adaptation  
genetic



ome

# A Genetic Explanation

- Suppose that “red head” is this Boolean function of 10 genes and “high temperature”

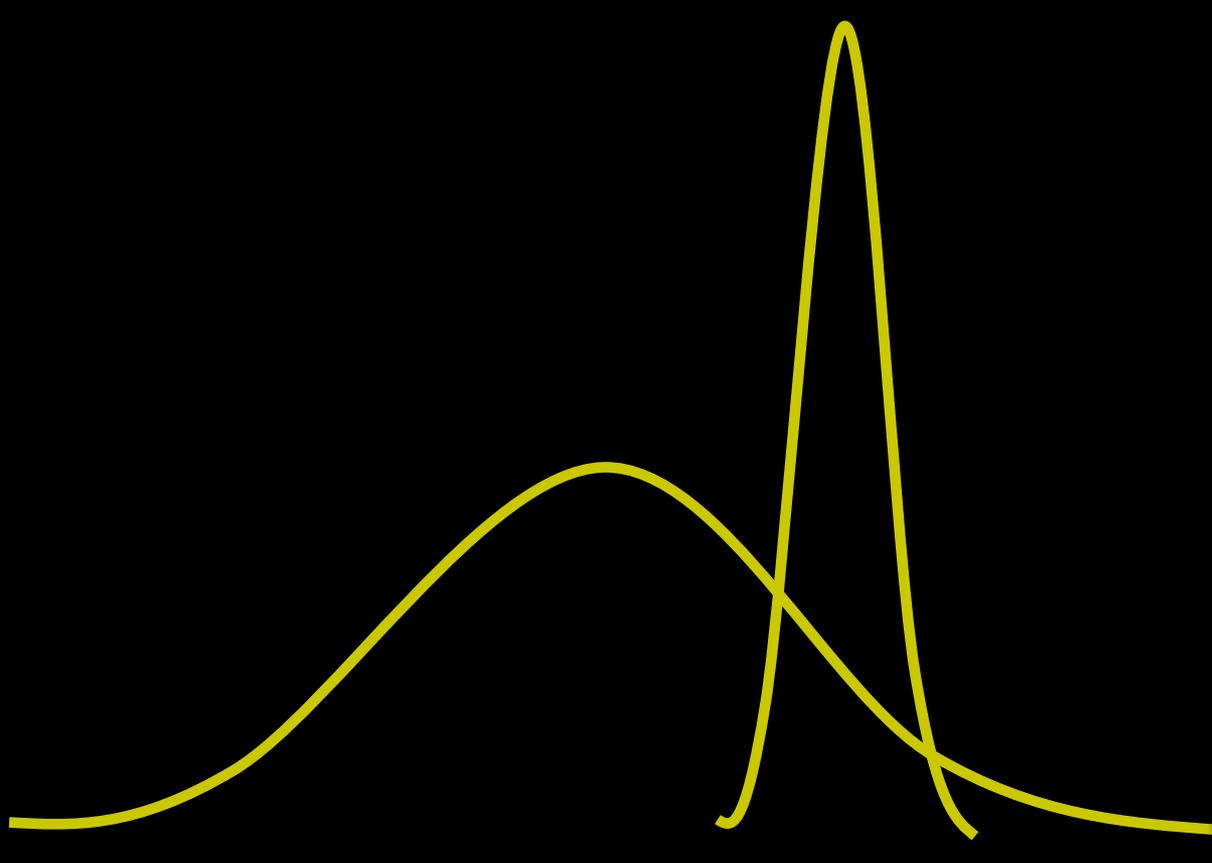
$$\text{“red head”} = \text{“}x_1 + x_2 + \dots + x_{10} + 3t \geq 10\text{”}$$

- Suppose also that the genes are independent random variables, with  $p_i$  initially half, say.

# A Genetic Explanation (cont.)

- In the beginning, no fly is red (the probability of being red is  $2^{-n}$ )
- With the help of  $t = 1$ , a few become red
- If you select them and breed them,  $\sim 60\%$  will be red!

Why 60%?



# A Genetic Explanation (cont.)

- Eventually, the population will be very biased towards  $x_i = 1$  (the  $p_i$ 's are close to 1)
- And so, a few flies will have all  $x_i = 1$  for all  $i$ , and they will stay red.

# Generalize!

- Let  $B$  is any Boolean function
- $n$  variables  $x_1 x_2 \dots x_n$  (no  $t$ )
- Independent, with probabilities

$$p = (p_1 p_2 \dots p_n)$$

- Now, generate a population of bit vectors, and select the ones that make  $B(x) = 1$

(cont.)

- In expectation,  $p \rightarrow p'$ ,  
where  $p_i' = \text{prob}_p(x_i = 1 \mid B(x) = 1)$

(Looking under the rug: Linkage?)

**Conjecture:** This solves SAT

**Theorem** (with Greg Valiant, 2011): If instead  $p \rightarrow \varepsilon p' + (1 - \varepsilon) p$ , then it converges to a satisfying assignment with prob. 1 (for large enough population and small enough  $\varepsilon$ ).

# Interpretation

- If there is any Boolean combination of a modestly large number of alleles that creates an unanticipated trait conferring even a small advantage, then this combination will be discovered and eventually fixed in the population.
- “With sex, all moderate-sized Boolean functions are evolvable.”

# Sooooo...

- The theory of life is deep and fascinating
- The point of view of a computer scientist makes it even more tantalizing
- Mixability helps understand the role of sex
- A natural stochastic process on Boolean functions may help illuminate genetic assimilation and the emergence of novel traits

Thank You!